EXAMINING FACTORIAL DESIGNS WITH STRUCTURAL EQUATION MODELING (SEM)

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Overview

- Factorial designs in ANOVA vs. SEM
- Multiple group models
- Steps for examining latent means
  - Nested model comparison
  - Testing interactions via contrast codes
- 3x2 Example
- Conclusions

Note. Presentation slides and example analyses are available online at http://crmda.ku.edu/presentations
Factorial Designs in ANOVA

- Normally distributed dependent variable (DV)
- Homogeneity of variances
  - Sphericity in repeated measures
- No measurement error
- Measurement invariance
Advantages of SEM

- Robust estimators can accommodate nonnormal data (Fan & Hancock, 2012)

- Model measurement error
  - Produces larger effect size

- Test measurement invariance across groups and/or time
  - Invariance is assumed in MIMIC models

- Similar or more power than MANOVA with multiple DVs (see Hancock, Lawrence, & Nevitt, 2000)
Multiple Group Models

- Specify a measurement model (i.e., CFA) for each group
  - If the design contains repeated measures, then only a model for each level of the between-subjects IV is needed

- Each model is estimated separately, but simultaneously
  - Model fit statistics are influenced by all groups

- Examine measurement invariance
  - Factor loading invariance (e.g., “weak invariance”)
  - Indicator intercept invariance (e.g., “strong invariance”)

### Example 2x2 Between Subjects

#### Multiple Group Model

<table>
<thead>
<tr>
<th>Factor B</th>
<th>A1</th>
<th>A2</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td><img src="image1" alt="Diagram" /></td>
<td><img src="image2" alt="Diagram" /></td>
</tr>
<tr>
<td></td>
<td>Group 1:</td>
<td>Group 2:</td>
</tr>
<tr>
<td></td>
<td><img src="image3" alt="Diagram" /></td>
<td><img src="image4" alt="Diagram" /></td>
</tr>
<tr>
<td>B2</td>
<td><img src="image5" alt="Diagram" /></td>
<td><img src="image6" alt="Diagram" /></td>
</tr>
<tr>
<td></td>
<td>Group 3:</td>
<td>Group 4:</td>
</tr>
<tr>
<td></td>
<td><img src="image7" alt="Diagram" /></td>
<td><img src="image8" alt="Diagram" /></td>
</tr>
</tbody>
</table>

- **Factor A**: $\eta$
- **Factor B**: $\eta$
- **Groups**: Group 1, Group 2, Group 3, Group 4
Steps for Examining Latent Means

- Omnibus Test
  - Constrain means of the same construct to be equal across all levels of each factor

- Main Effects
  - Factor A: Constrain means equal across levels of Factor A
  - Factor B: Constrain means equal across levels of Factor B

- TIP: Check $df$ for the constrained model to ensure the constraints were imposed.
Nested Model Comparisons

- A model with latent means constrained to be equal is considered “nested” within the strong invariance model where the means are freely estimated.
- Nested models can be compared via a $\chi^2$ difference test*, where:
  - $\Delta \chi^2 = \chi^2_{\text{Constrained}} - \chi^2_{\text{Strong invariance}}$
  - $\Delta df = df_{\text{Constrained}} - df_{\text{Strong invariance}}$
- If $\Delta \chi^2$ is significant, then the model constraint is NOT supported.

*Note. The above holds for maximum likelihood (ML) estimation. If robust maximum likelihood or weighted least squares estimators are used, the scaling factor must be incorporated (see Satorra & Bentler, 2001).
Examining Factorial Interaction

- Omnibus and main effect tests of latent means ignore a possible Factor A x Factor B interaction.

- Similar to ANOVA, orthogonal interaction contrast codes can be used as model constraints on latent means.

- Each contrast adds one degree of freedom ($df$).
Examining Factorial Interaction

- Number of necessary contrasts $= (J-1)(K-1)$, where:
  - $J =$ # levels in Factor A
  - $K =$ # levels in Factor B

- To test for interaction, estimate a model with a full set of interaction contrasts and compare to the strong invariance model via $\chi^2$ difference test
## Orthogonal Interaction Contrast Codes

- **2x2 Design**
  
<table>
<thead>
<tr>
<th>Factor A</th>
<th>A1</th>
<th>A2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor B</td>
<td>B1</td>
<td>B2</td>
</tr>
<tr>
<td>Int. Contrast 1</td>
<td>1</td>
<td>-1</td>
</tr>
</tbody>
</table>

  Note. Remember to set each contrast to equal 0 in your model.

- **2x3 Design**
  
<table>
<thead>
<tr>
<th>Factor A</th>
<th>A1</th>
<th>A2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor B</td>
<td>B1</td>
<td>B2</td>
</tr>
<tr>
<td>Int. Contrast 1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Int. Contrast 2</td>
<td>1</td>
<td>-2</td>
</tr>
</tbody>
</table>

- **3x3 Design**
  
<table>
<thead>
<tr>
<th>Factor A</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Int. Contrast 1</td>
<td>1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>Int. Contrast 2</td>
<td>1</td>
<td>-2</td>
<td>1</td>
</tr>
<tr>
<td>Int. Contrast 3</td>
<td>1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>Int. Contrast 4</td>
<td>1</td>
<td>-2</td>
<td>1</td>
</tr>
</tbody>
</table>
A 3x2 Mixed Design Example

- Does undergraduate students’ Creationist Reasoning change after completing a college science course?
- Examined three courses at the beginning and end of the semester
  - Intro Biology for Biology Majors ($N = 631$)
  - Evolutionary Psychology ($N = 65$)
  - Intro to U.S. Politics ($N = 366$) Control Group
Creationist Reasoning (CR) (see Hawley et al., 2011)
- Items on 1(Strongly Disagree) to 7(Strongly Agree) scale
- Higher responses = higher CR

CR is comprised of several subscales
- Intelligent design fallacies (IDF)
- Young-earth creationism (YEC)
- Moral objections (MO)
- Social objections (SO)
- Distrust of the scientific enterprise (DSE)
Example: Creationist Reasoning CFA

Note. Indicator residuals are correlated for repeated measures.
Example: Multiple Group CFA

Group 1: Intro U.S. Politics

Group 2: Intro Bio

Group 3: Evo Psyc
Example: Creationist Reasoning Latent Means

<table>
<thead>
<tr>
<th></th>
<th>Political Science</th>
<th>Biology</th>
<th>Evolutionary Psychology</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Time 1</strong></td>
<td>3.216</td>
<td>3.060</td>
<td>2.372</td>
</tr>
<tr>
<td>(0.922)</td>
<td>(0.989)</td>
<td>(0.843)</td>
<td></td>
</tr>
<tr>
<td><strong>Time 2</strong></td>
<td>3.219</td>
<td>3.079</td>
<td>2.027</td>
</tr>
<tr>
<td>(1.001)</td>
<td>(0.994)</td>
<td>(0.677)</td>
<td></td>
</tr>
</tbody>
</table>

*Note. Means (standard deviations). Effects-coding method of model identification was used to maintain the original scale of the indicators (see Little, Slegers, & Card, 2006)*
Example: Mplus Code for Interaction

MODEL:
!Time 1;
CreaReas BY IDF* (L1)
   YEC (L2)
   MoralObj (L3)
   SocObj (L4)
   DisTrust (L5);

!Time 2 equate loadings for weak invariance;
ZCreaReas BY ZIDF* (L1)
   ZYEC (L2)
   ZMoralObj (L3)
   ZSocObj (L4)
   ZDistrust (L5);

!Correlate Time1 & Time2 Residuals;
IDF WITH ZIDF;
YEC WITH ZYEC;
MoralObj WITH ZMoralObj;
SocObj WITH ZSocObj;
DisTrust WITH ZDistrust;

!Correlate Time1 & Time2 Residuals;
IDF WITH ZIDF;
YEC WITH ZYEC;
MoralObj WITH ZMoralObj;
SocObj WITH ZSocObj;
DisTrust WITH ZDistrust;

!Create interaction contrast codes;
NEW (C1 C2);
C1 = A1-A3-A4+A6;

MODEL Pols:
!Label means for model constraint;
[CreaReas] (A1);
[ZCreaReas] (A2);

MODEL Bio:
[CreaReas] (A3);
[ZCreaReas] (A4);

MODEL EvoPsyc:
[CreaReas] (A5);
[ZCreaReas] (A6);
## Example: Test of Latent Means Results

<table>
<thead>
<tr>
<th>Model</th>
<th>$\chi^2$</th>
<th>df</th>
<th>$p$</th>
<th>$\Delta \chi^2$</th>
<th>$\Delta$ df</th>
<th>$p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strong Invariance</td>
<td>586.72</td>
<td>127</td>
<td>&lt;.001</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Latent Mean Invariance (Omnibus)</td>
<td>683.18</td>
<td>132</td>
<td>&lt;.001</td>
<td>96.46</td>
<td>5</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>Interaction (via contrast codes)</td>
<td>630.11</td>
<td>129</td>
<td>&lt;.001</td>
<td>43.39</td>
<td>2</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>Group</td>
<td>682.62</td>
<td>131</td>
<td>&lt;.001</td>
<td>95.90</td>
<td>4</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>Time</td>
<td>615.04</td>
<td>130</td>
<td>&lt;.001</td>
<td>28.32</td>
<td>3</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>Pols</td>
<td>586.72</td>
<td>128</td>
<td>&lt;.001</td>
<td>0.004</td>
<td>1</td>
<td>0.950</td>
</tr>
<tr>
<td>Bio</td>
<td>587.14</td>
<td>128</td>
<td>&lt;.001</td>
<td>0.42</td>
<td>1</td>
<td>0.519</td>
</tr>
<tr>
<td>Evo Psyc</td>
<td>614.62</td>
<td>128</td>
<td>&lt;.001</td>
<td>27.90</td>
<td>1</td>
<td>&lt;.001</td>
</tr>
</tbody>
</table>

- Significant Group X Time interaction, $\Delta \chi^2 (2) = 43.39$, $p < .001$
- Significant simple main effect for Evolutionary Psychology course $\Delta \chi^2 (1) = 27.90$, $p < .001$
  - CR significantly decreased from Time 1 ($A = 2.37$) to Time 2 ($A = 2.03$)
Example: Creationist Reasoning Interaction

- Political Science
- Biology
- Evolutionary Psychology
Disadvantages

- More difficult to implement than ANOVA
- Large factorial designs require many contrasts
  - \((J-1)(K-1)\) contrasts needed (same \(df\) as interaction in ANOVA)
- Requires larger sample sizes
Additional Advantages

- Can be used for between, within, or mixed subjects designs
- Multiple dependent variables (DVs)
  - e.g., MANOVA designs
- Covariates
  - e.g., ANCOVA or MANCOVA designs
- Allows for more complex hypotheses
  - e.g., moderated mediation
- May provide greater statistical power over MANOVA (see Hancock, Lawrence, & Nevitt, 2000)
Thank You

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- Presentation slides and example analyses are available online at http://crmda.ku.edu/presentations
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